

## 11.4

## Arcs and Chords

## Goal

Use properties of chords of circles.

## Key Words

- congruent arcs p. 602
- perpendicular bisector p. 274

By finding the perpendicular bisectors of two chords, an archaeologist can recreate a whole plate from just one piece.

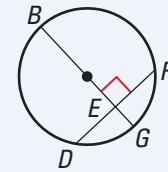
This approach relies on Theorem 11.5, and is shown in Example 2.



## THEOREM 11.4

**Words** If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

**Symbols** If  $\overline{BG} \perp \overline{FD}$ , then  $\overline{DE} \cong \overline{EF}$  and  $\widehat{DG} \cong \widehat{GF}$ .



## EXAMPLE 1 Find the Length of a Chord

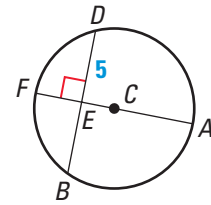
In  $\odot C$  the diameter  $\overline{AF}$  is perpendicular to  $\overline{BD}$ . Use the diagram to find the length of  $\overline{BD}$ .

## Solution

Because  $\overline{AF}$  is a diameter that is perpendicular to  $\overline{BD}$ , you can use Theorem 11.4 to conclude that  $\overline{AF}$  bisects  $\overline{BD}$ . So,  $BE = ED = 5$ .

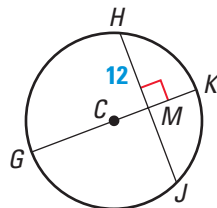
$$\begin{aligned} BD &= BE + ED && \text{Segment Addition Postulate} \\ &= 5 + 5 && \text{Substitute 5 for } BE \text{ and } ED. \\ &= 10 && \text{Simplify.} \end{aligned}$$

**ANSWER** ▶ The length of  $\overline{BD}$  is 10.

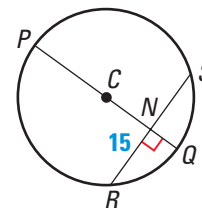


## Checkpoint Find the Length of a Segment

1. Find the length of  $\overline{JM}$ .



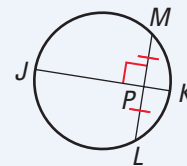
2. Find the length of  $\overline{SR}$ .



**THEOREM 11.5**

**Words** If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

**Symbols** If  $\overline{JK} \perp \overline{ML}$  and  $\overline{MP} \cong \overline{PL}$ , then  $\overline{JK}$  is a diameter.



All diameters of a circle include the center of the circle. Therefore, the point where two diameters intersect is the center of the circle.

### Link to Careers



**ARCHAEOLOGISTS** study and reconstruct artifacts which provide information about past cultures.

**EXAMPLE 2 Find the Center of a Circle**

Suppose an archaeologist finds part of a circular plate. Show how to reconstruct the original shape of the plate.

**Solution**

- 1 Draw any two chords that are not parallel to each other.
- 2 Draw the perpendicular bisector of each chord. These lines contain diameters.
- 3 The diameters intersect at the circle's center. Use a compass to draw the rest of the plate.

**Student Help****STUDY TIP**

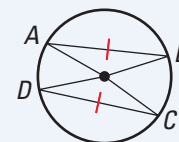
If two central angles are congruent then their corresponding arcs are congruent.

**THEOREM 11.6**

**Words** In the same circle, or in congruent circles:

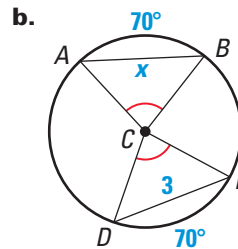
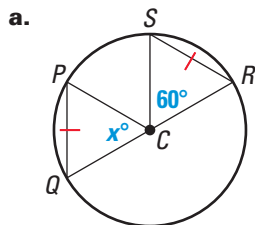
- If two chords are congruent, then their corresponding minor arcs are congruent.
- If two minor arcs are congruent, then their corresponding chords are congruent.

**Symbols** If  $\overline{AB} \cong \overline{DC}$ , then  $\widehat{AB} \cong \widehat{DC}$ .  
If  $\widehat{AB} \cong \widehat{DC}$ , then  $\overline{AB} \cong \overline{DC}$ .



**EXAMPLE 3 Find Measures of Angles and Chords**

Find the value of  $x$ .



**Solution**

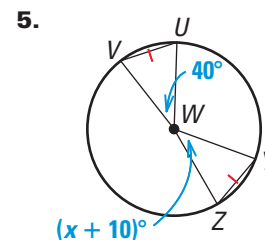
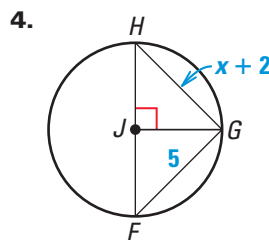
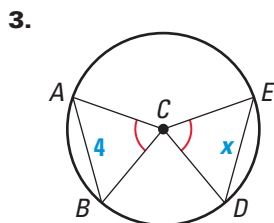
a. Because  $\overline{QP} \cong \overline{RS}$ , it follows that  $\widehat{QP} \cong \widehat{RS}$ .

So,  $m\widehat{QP} = m\widehat{RS} = 60^\circ$ , and  $x = 60$ .

b. Because  $\overline{AB} \cong \overline{DE}$ , it follows that  $\widehat{AB} \cong \widehat{DE}$ . So,  $x = DE = 3$ .

**Checkpoint** Find Measures of Angles and Chords

Find the value of  $x$ .

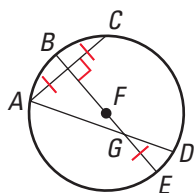


**11.4 Exercises**

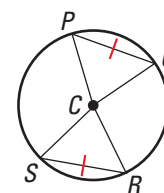
**Guided Practice**

**Vocabulary Check**

1. Identify a diameter.

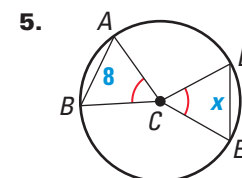
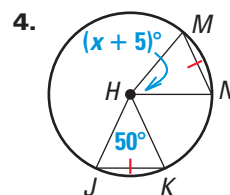
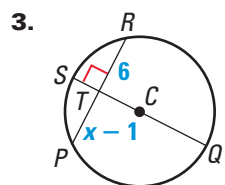


2. Identify a pair of congruent arcs.



**Skill Check**

Find the value of  $x$ .

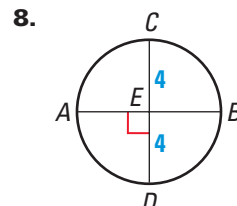
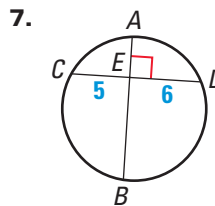
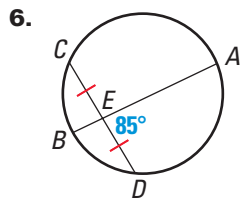


# Practice and Applications

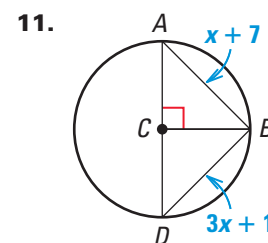
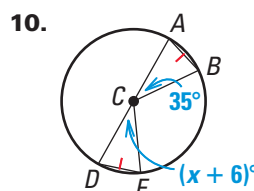
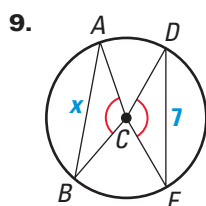
## Extra Practice

See p. 695.

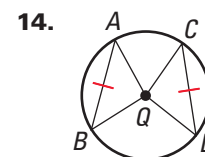
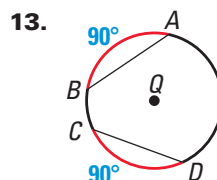
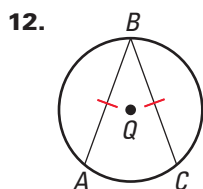
**Identifying Diameters** Determine whether  $\overline{AB}$  is a diameter of the circle. Explain your reasoning.



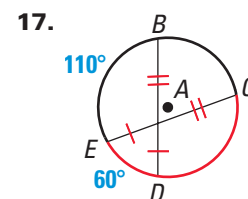
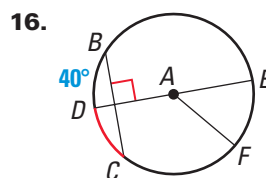
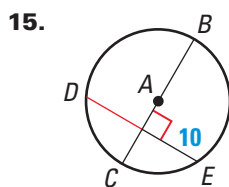
**Finding Chords and Central Angles** Find the value of  $x$ .



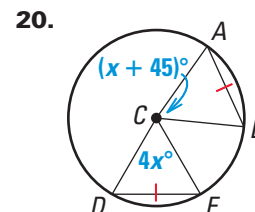
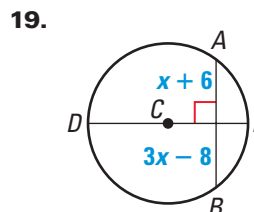
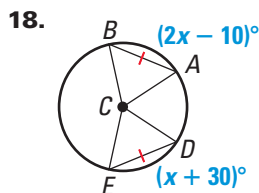
**Logical Reasoning** Name any congruent arcs, chords, or angles. State a postulate or theorem that justifies your answer.



**Finding Measures** Find the measure of the red segment or arc.



**Using Algebra** Find the value of  $x$ .



### Homework Help

**Example 1:** Exs. 6–8, 15–20

**Example 2:** Exs. 21–22

**Example 3:** Exs. 9–20

21. **Visualize It!** Draw a large circle and cut it out. Tear part of it off and ask another student to recreate your circle.

**Link to Careers**



**EMTs** Some Emergency Medical Technicians (EMTs) train specifically for wilderness emergencies.

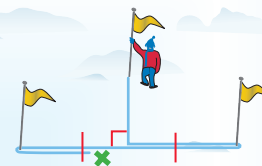


**22. Avalanche Rescue Beacon** An avalanche rescue beacon is a device used by backcountry skiers. It gives off a signal that is detectable within a circle of a certain radius. In a practice drill, a ski patrol uses the following steps to locate a beacon buried in the snow. Explain how it works.

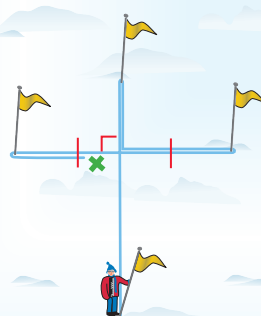
- 1 Walk in a straight line until the signal disappears. Turn around and walk back until the signal disappears again.



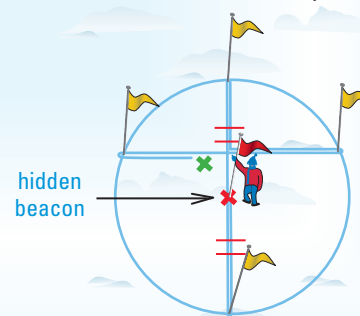
- 2 Walk back to the halfway point, and walk away from the line at a 90° angle until the signal disappears.



- 3 Turn around and walk in a straight line until the signal disappears again.



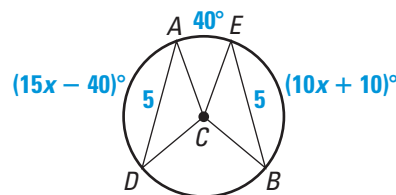
- 4 Walk back to the halfway point. You will be near the center of the circle. The beacon is under you.



**Standardized Test Practice**

**23. Multi-Step Problem** Use the diagram below.

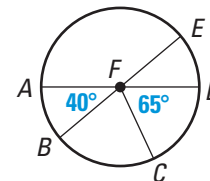
- a. Explain why  $\widehat{AD} \cong \widehat{BE}$ .
- b. Find the value of  $x$ .
- c. Find  $m\widehat{AD}$  and  $m\widehat{BE}$ .
- d. Find  $m\widehat{BD}$ .



**Mixed Review**

**Measuring Arcs** In the diagram below,  $\overline{AD}$  and  $\overline{BE}$  are diameters of  $\odot F$ . Find the measure. (Lesson 11.3)

- |                      |                      |
|----------------------|----------------------|
| 24. $m\widehat{DE}$  | 25. $m\widehat{BC}$  |
| 26. $m\widehat{AE}$  | 27. $m\widehat{BCD}$ |
| 28. $m\widehat{ABC}$ | 29. $m\widehat{ADE}$ |



**Algebra Skills**

**Comparing Numbers** Compare the two numbers. Write the answer using  $<$ ,  $>$ , or  $=$ . (Skills Review, p. 662)

- |                     |  |                             |
|---------------------|--|-----------------------------|
| 30. $-26$ and $-29$ | 31. $\frac{15}{20}$ and $-\frac{3}{4}$ | 32. $0.2$ and $\frac{1}{5}$ |
|---------------------|--|-----------------------------|