Product to Sum

Sum to Product

$\cos(x+y) + \cos(x-y)$

$\cos x \cos y - \cancel{\sin x \sin y} + \cos x \cos y + \cancel{\sin x \sin y}$

$\frac{2\cos x \cos y}{2} = \frac{\cos(x+y) + \cos(x-y)}{2}$

$$\boxed{\cos x \cos y = \frac{1}{2}\left(\cos(x+y) + \cos(x-y)\right)}$$

$$\cos(x-y) - \cos(x+y)$$

$$\cancel{\cos x \cos y} + \sin x \sin y - [\cancel{\cos x \cos y} - \sin x \sin y]$$

$$\boxed{\sin x \sin y = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)}$$

$$\sin(x+y) + \sin(x-y)$$

$$\sin x\cos y + \cancel{\cos x\sin y} + \sin x\cos y - \cancel{\cos x\sin y}$$

$$\boxed{\sin x\cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]}$$

$$\underbrace{\sin(x+y)+\sin(x-y)} = 2\sin x\cos y$$

$$x+y=A$$

$$-(x-y=B)$$

$$2y = A-B$$

$$\boxed{\sin A+\sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$x=\frac{A+B}{2} \qquad y=\frac{A-B}{2}$$

$$\sin(x+y) + \sin(x-y)$$

$$\sin x\cos y + \cancel{\cos x \sin y} + \sin x \cos y - \cancel{\cos x \sin y}$$

$$\boxed{\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]}$$

Ex 1

$$\cos\left(\frac{13\pi}{12}\right)\sin\frac{5\pi}{12} = \frac{1}{2}\left(\sin\left(\underset{}{\overset{\frac{3\pi}{2}}{\frac{18\pi}{12}}}\right) - \sin\left(\overset{\frac{2\pi}{3}}{\frac{8\pi}{12}}\right)\right)$$

$$= \frac{1}{2}\left(-1 - \frac{\sqrt{3}}{2}\right)$$

$$= -\frac{1}{2} - \frac{\sqrt{3}}{4} = \frac{-2-\sqrt{3}}{4}$$

Ex 2

$\Longrightarrow \sin \cancel{4x} - \sin 2x = 0$

$= \cancel{2} \cos 3x \sin x = 0$

$\cos 3x = 0$ $\qquad$ $\sin x = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ $\qquad$ $0, \pi, 2\pi, 3\pi \ldots$

$\frac{3x}{3} = \frac{\frac{\pi}{2}}{3} + \frac{\pi \cdot n}{3}$ $\qquad$ $x = n \cdot \pi$

$x = \frac{\pi}{6} + \frac{\pi}{3} \cdot n$